

Section 4.3: Quadratic functions and their properties

#1-12: For each problem do the following

1) $f(x - 3) + 4$

a) Find the indicated function and describe the transformation as compared to the function $f(x) = x^2$, specifically state if the graph is shifted left, right, up, down and if any reflection has occurred

$f(x-3)+4 = (x-3)^2 + 4$

x^2 TABLE

-2	4
-1	1
0	0
1	1
2	4

$(x-3)^2 + 4$
 ADD 3 TO EACH X
 ADD 4 TO EACH Y

b) make a table of values and sketch a graph.

c) state the domain and range of the function

ALL PARABOLA Domain $(-\infty, \infty)$ Range $[4, \infty)$

d) state the intervals where the function is increasing and decreasing

INC $(3, \infty)$ DEC $(-\infty, 3)$

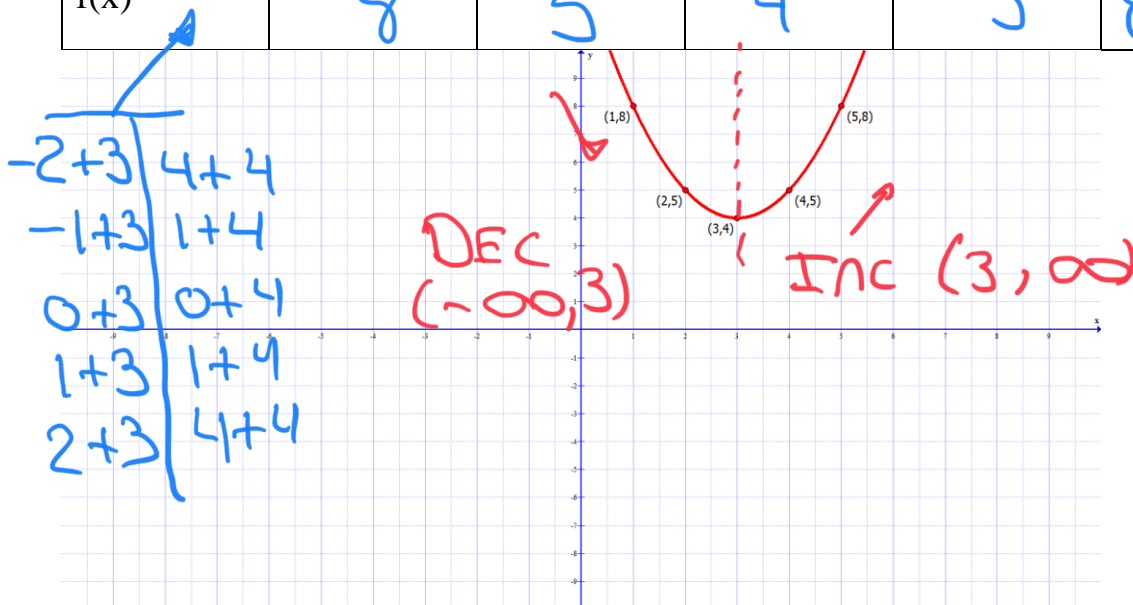
e) state if the function has a local maximum point, if it does state the local maximum value

NO MAX

f) state if the function has a local minimum point, if it does state the local minimum value

LOCAL MINIMUM POINT, L.mi. $y=4$ when $x=3$

x	1	2	3	4	5
f(x)	8	5	4	5	8



3) $2f(x+3) - 4$

a) Find the indicated function and describe the transformation as compared to the function $f(x) = x^2$, specifically state if the graph is shifted left, right, up, down and if any reflection has occurred

$2f(x+3) - 4 = 2(x+3)^2 - 4$

b) make a table of values and sketch a graph.

See Below

↑ Stretched, left 3, Down 4

c) state the domain and range of the function

Domain $(-\infty, \infty)$ Range $[-4, \infty)$

d) state the intervals where the function is increasing and decreasing

Dec. $(-\infty, -3)$ Inc. $(-3, \infty)$

e) state if the function has a local maximum point, if it does state the local maximum value

None

f) state if the function has a local minimum point, if it does state the local minimum value

Local Minimum point $(-3, -4)$
 L.M.:V. $y = -4$ when $x = -3$

x	-5	-4	-3	-2	-1
f(x)	-4	-2	-4	-2	-4

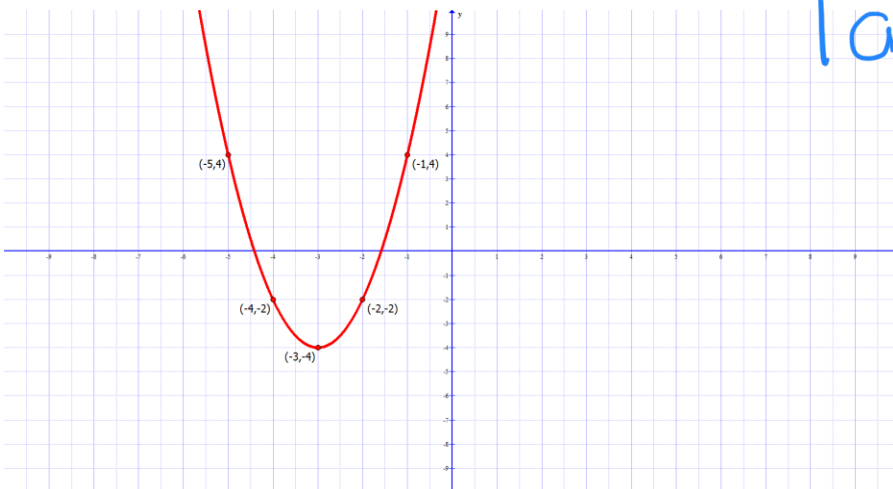


Table $2(x+3)^2 - 4$

↑
Change Sign

$x = -3$
middle

x-column

$$5) \frac{1}{2}f(x+4) - 6$$

a) Find the indicated function and describe the transformation as compared to the function $f(x) = x^2$, specifically state if the graph is shifted left, right, up, down and if any reflection has occurred

$$\frac{1}{2}f(x+4) - 6 = \frac{1}{2}(x+4)^2 - 6$$

b) make a table of values and sketch a graph.

compressed, left 4, down 6

c) state the domain and range of the function

Domain $(-\infty, \infty)$ Range $[-6, \infty)$

d) state the intervals where the function is increasing and decreasing

Inc. $(-4, \infty)$ Dec $(-\infty, -4)$

e) state if the function has a local maximum point, if it does state the local maximum value

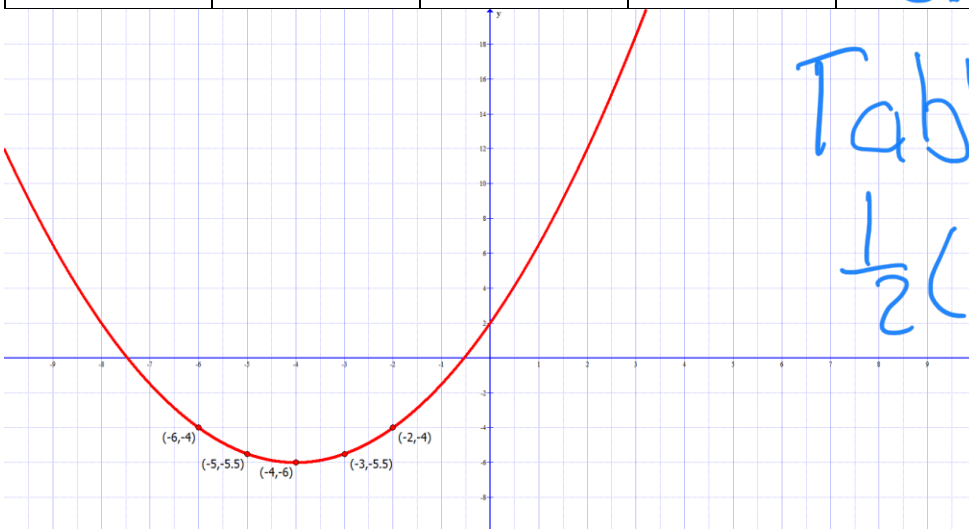
None

f) state if the function has a local minimum point, if it does state the local minimum value

Local Minimum Point $(-4, -6)$

L.Mi. v $y = -6$ when $x = -4$

x	-6	-5	-4	-3	-2
f(x)	-4	-5.5	-6	-5.5	-4



Table

$$\frac{1}{2}(x+4)^2 - 6$$

change sign

$x = -4$ middle

x-column

7) $-2f(x) + 3$

a) Find the indicated function and describe the transformation as compared to the function $f(x) = x^2$, specifically state if the graph is shifted left, right, up, down and if any reflection has occurred

$-2f(x) + 3 = -2x^2 + 3$

b) make a table of values and sketch a graph. See below

c) state the domain and range of the function

Domain $(-\infty, \infty)$ Range $(-\infty, 3]$

d) state the intervals where the function is increasing and decreasing

Inc. $(-\infty, 0)$ Dec $(0, \infty)$

e) state if the function has a local maximum point, if it does state the local maximum value

Local maximum point $(0, 3)$
L. Ma. V $y = 3$ when $x = 0$

f) state if the function has a local minimum point, if it does state the local minimum value

None

x	-2	-1	0	1	2
f(x)	-5	1	3	1	-5

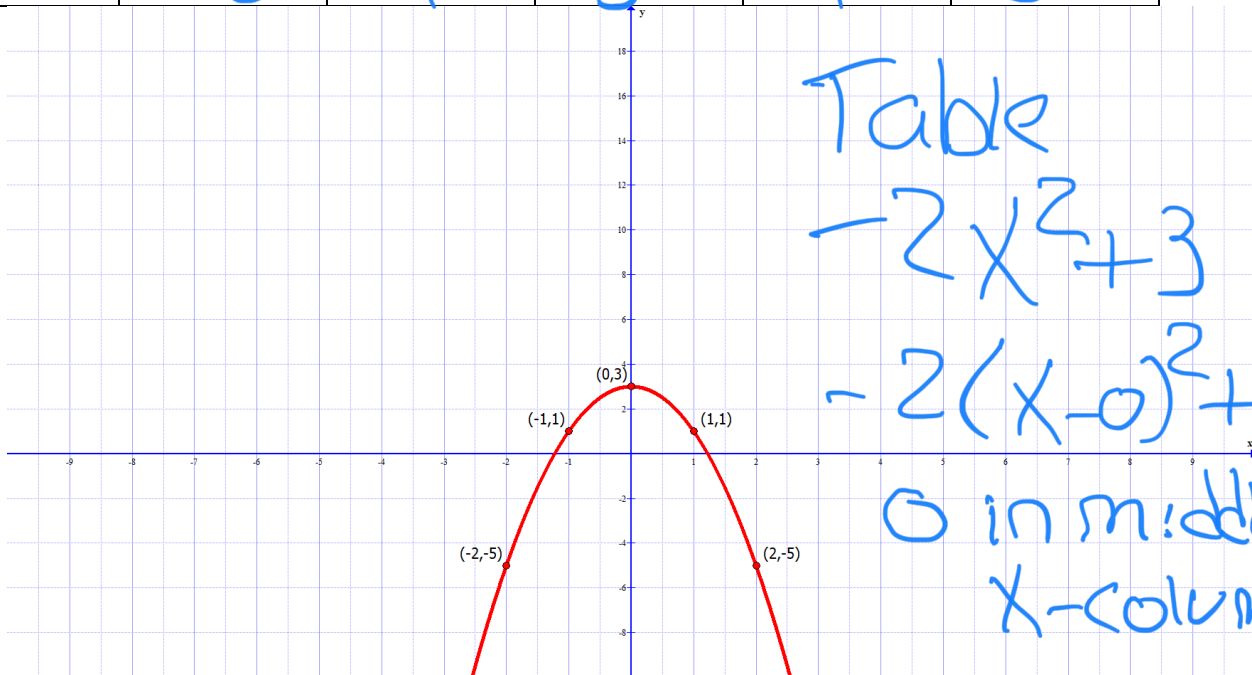


Table
 $-2x^2 + 3$
 $-2(x-0)^2 + 3$
 0 in middle
 x-column

$$9) -\frac{1}{4}f(x+5) - 2$$

a) Find the indicated function and describe the transformation as compared to the function $f(x) = x^2$, specifically state if the graph is shifted left, right, up, down and if any reflection has occurred

$$-\frac{1}{4}f(x+5) - 2 = -\frac{1}{4}(x+5)^2 - 2$$

b) make a table of values and sketch a graph.

REFLECT X-AXIS compress left 5 down 2

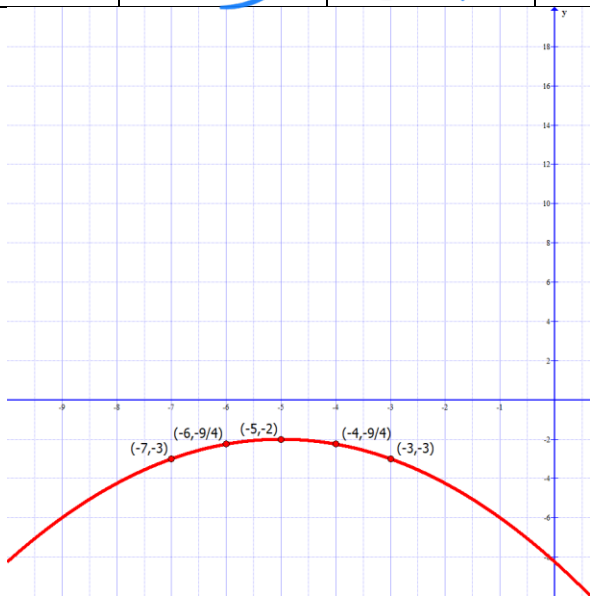
c) state the domain and range of the function

d) state the intervals where the function is increasing and decreasing

e) state if the function has a local maximum point, if it does state the local maximum value

f) state if the function has a local minimum point, if it does state the local minimum value

x	-7	-6	-5	-4	-3
f(x)	-3	-2.25	-2	-2.25	-3



Table

$$-\frac{1}{4}(x+5)^2 - 2$$

Change Sign

$$x = -5$$

middle

x-column

11) $2f(x+3)+4$

a) Find the indicated function and describe the transformation as compared to the function $f(x) = x^2$, specifically state if the graph is shifted left, right, up, down and if any reflection has occurred

$2f(x+3)+4 = 2(x+3)^2 + 4$
 ↑ stretches left 3 up 4

b) make a table of values and sketch a graph.

See below

c) state the domain and range of the function

Domain $(-\infty, \infty)$ Range $[4, \infty)$

d) state the intervals where the function is increasing and decreasing

Inc. $(-3, \infty)$ Dec. $(-\infty, -3)$

e) state if the function has a local maximum point, if it does state the local maximum value

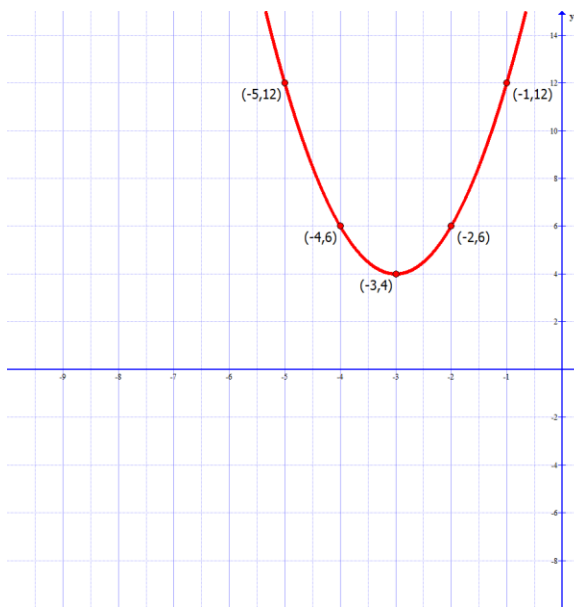
None.

f) state if the function has a local minimum point, if it does state the local minimum value

Local minimum point $(-3, 4)$

Local minimum value $y=4$, when $x=-3$

x	-5	-4	-3	-2	-1
f(x)	12	6	4	6	12



Table

$2(x+3)^2 + 4$



Change Sign

$x = -4$

middle

x-column

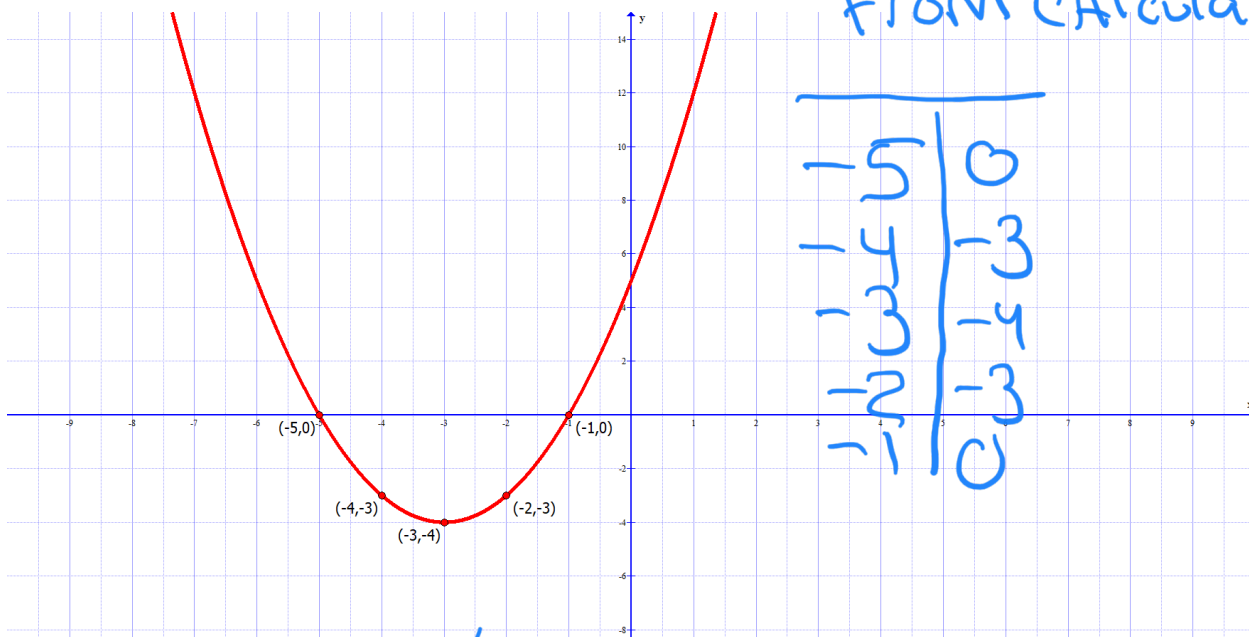
#13 – 24: For each problem do the following:

13) $f(x) = x^2 + 6x + 5$

a) Use completing the square to rewrite the problem in standard form

b) Describe the transformation as compared to the function $f(x) = x^2$ *left 3 Down 4*

c) Sketch a graph, make sure to label the vertex. You may use your calculator, instead of making a table of values to create your graph



from calculator

$$\begin{array}{r|l} -5 & 0 \\ -5 & -3 \\ -3 & -4 \\ -1 & -3 \\ 1 & 0 \end{array}$$

a) $f(x) = (x^2 + 6x + c) + 5 - 3$

$$c = \left(\frac{6}{2}\right)^2 = (3)^2 = 9$$

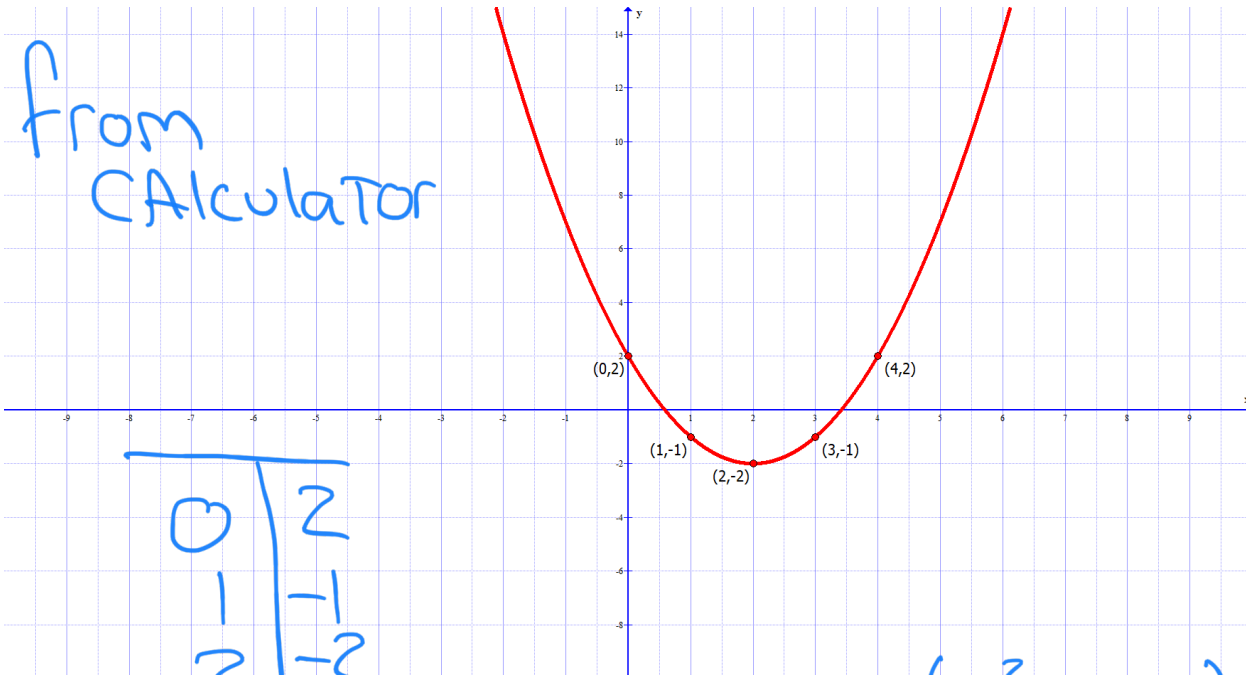
$$f(x) = (x^2 + 6x + 9) + 5 - 9$$

$$\boxed{f(x) = (x+3)^2 - 4}$$

15) $k(x) = x^2 - 4x + 2$

- a) Use completing the square to rewrite the problem in standard form
- b) Describe the transformation as compared to the function $f(x) = x^2$
- c) Sketch a graph, make sure to label the vertex. You may use your calculator, instead of making a table of values to create your graph

from
Calculator



$$\begin{array}{r} 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \overline{) \begin{array}{r} 2 \\ -1 \\ -2 \\ -1 \\ 2 \end{array}}$$

15a) $k(x) = (x^2 - 4x + c) + 2 - c$
 $c = (-\frac{4}{2})^2 = (-2)^2 = 4$

$k(x) = (x^2 - 4x + 4) + 2 - 4$

$k(x) = (x - 2)^2 - 2$

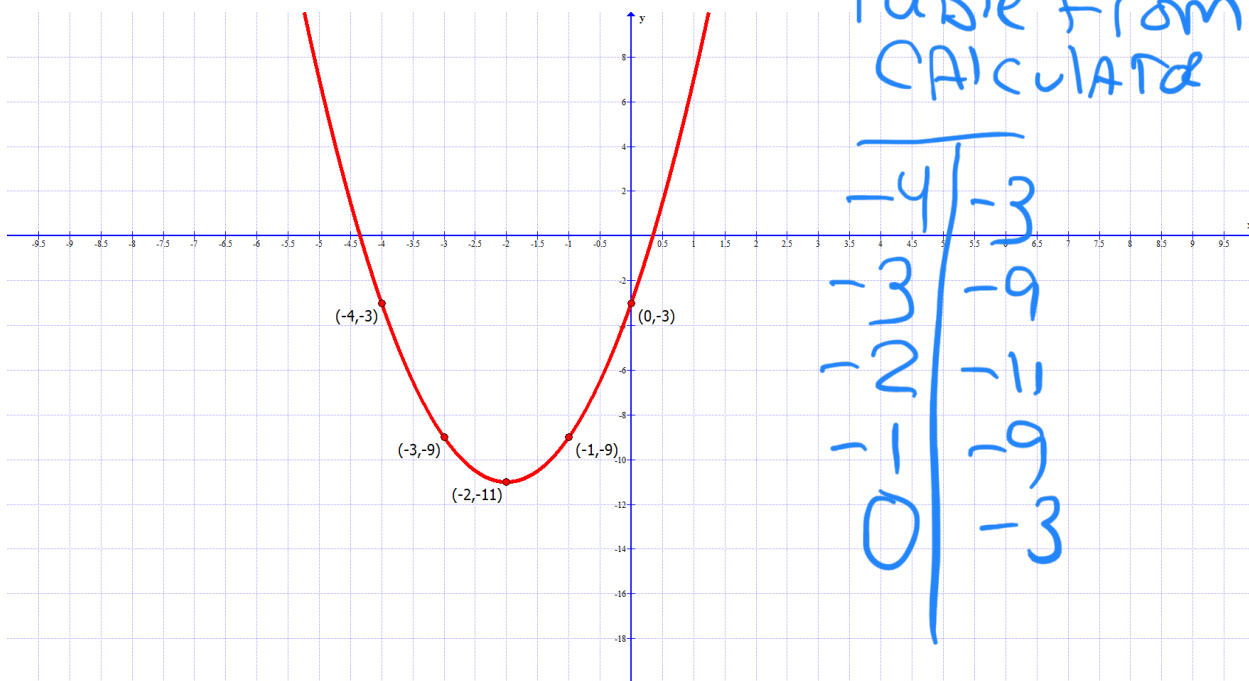
17) $f(x) = 2x^2 + 8x - 3$

a) Use completing the square to rewrite the problem in standard form

b) Describe the transformation as compared to the function $f(x) = x^2$

Stretched, left 2, Down 11

c) Sketch a graph, make sure to label the vertex. You may use your calculator, instead of making a table of values to create your graph



17a) $f(x) = 2(x^2 + 4x + c) - 3 - 2c$

$c = (\frac{4}{2})^2 = (2)^2 = 4$

$f(x) = 2(x^2 + 4x + 4) - 3 - 2(4)$

$= 2(x+2)^2 - 3 - 8$

$= 2(x+2)^2 - 11$

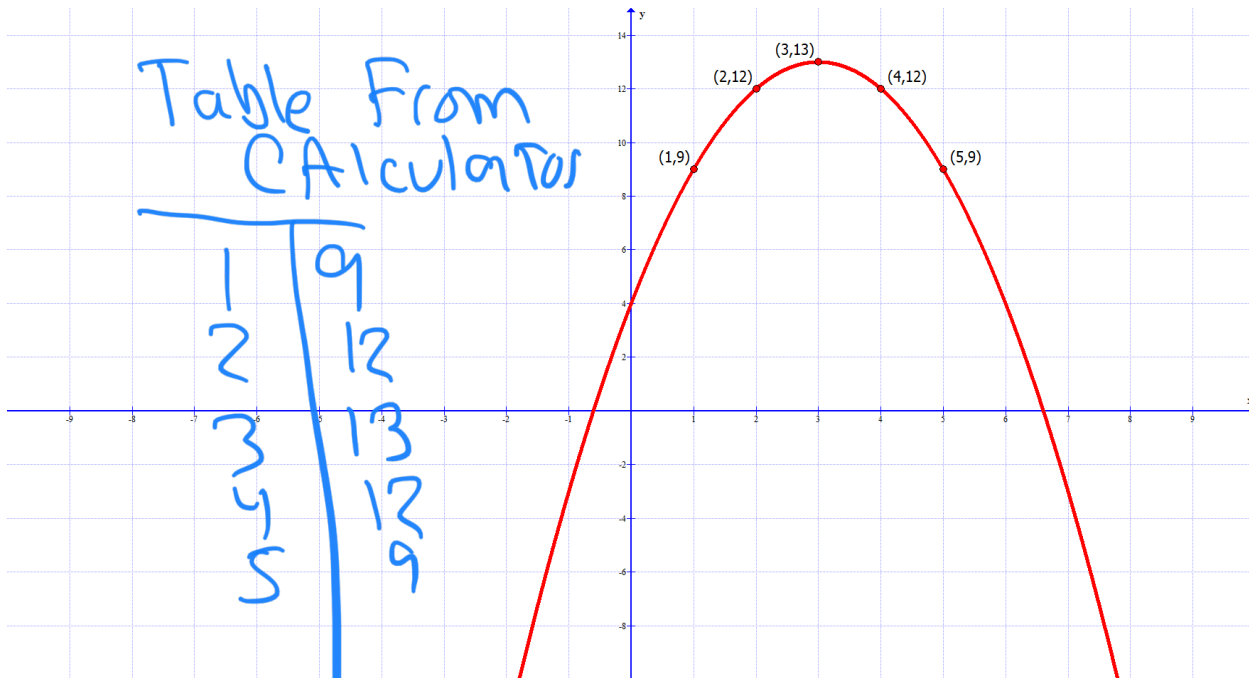
19) $f(x) = -x^2 + 6x + 4$

a) Use completing the square to rewrite the problem in standard form

b) Describe the transformation as compared to the function $f(x) = x^2$

reflect x-axis, Right 3, up 13

c) Sketch a graph, make sure to label the vertex. You may use your calculator, instead of making a table of values to create your graph



19a) $f(x) = -1(x^2 - 6x + c) + 4 + 1c$

$c = (-\frac{6}{2})^2 = (-3)^2 = 9$

$f(x) = -1(x^2 - 6x + 9) + 4 + 1(9)$

$f(x) = -1(x-3)^2 + 13$

$f(x) = -(x-3)^2 + 13$

21) $k(x) = -2x^2 + 12x - 7$

a) Use completing the square to rewrite the problem in standard form

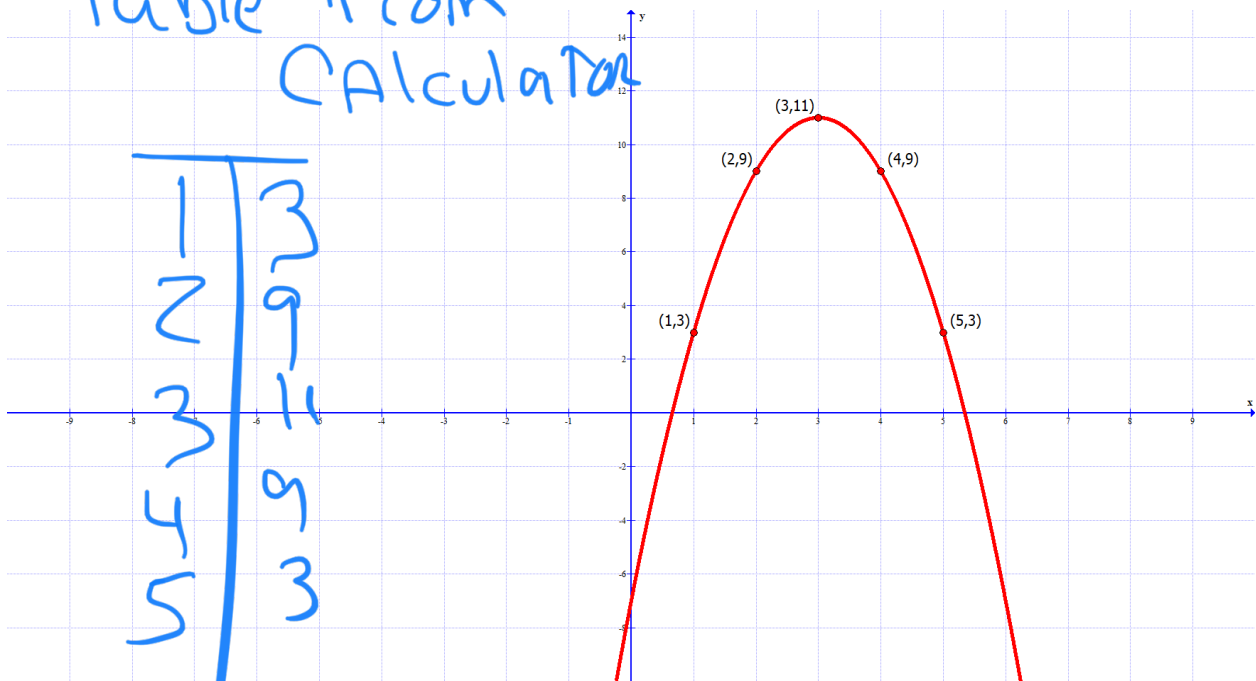
b) Describe the transformation as compared to the function $f(x) = x^2$

reflect x-axis, Stretched, Right 3 up 11

c) Sketch a graph, make sure to label the vertex. You may use your calculator, instead of making a table of values to create your graph

Table from Calculator

1	3
2	9
3	11
4	9
5	3



21a) $f(x) = -2(x^2 - 6x + c) - 7 + 2c$
 $c = (-6/2)^2 = (-3)^2 = 9$

$f(x) = -2(x-3)^2 - 7 + 2(9)$

$f(x) = -2(x-3)^2 - 7 + 18$

$f(x) = -2(x-3)^2 + 11$

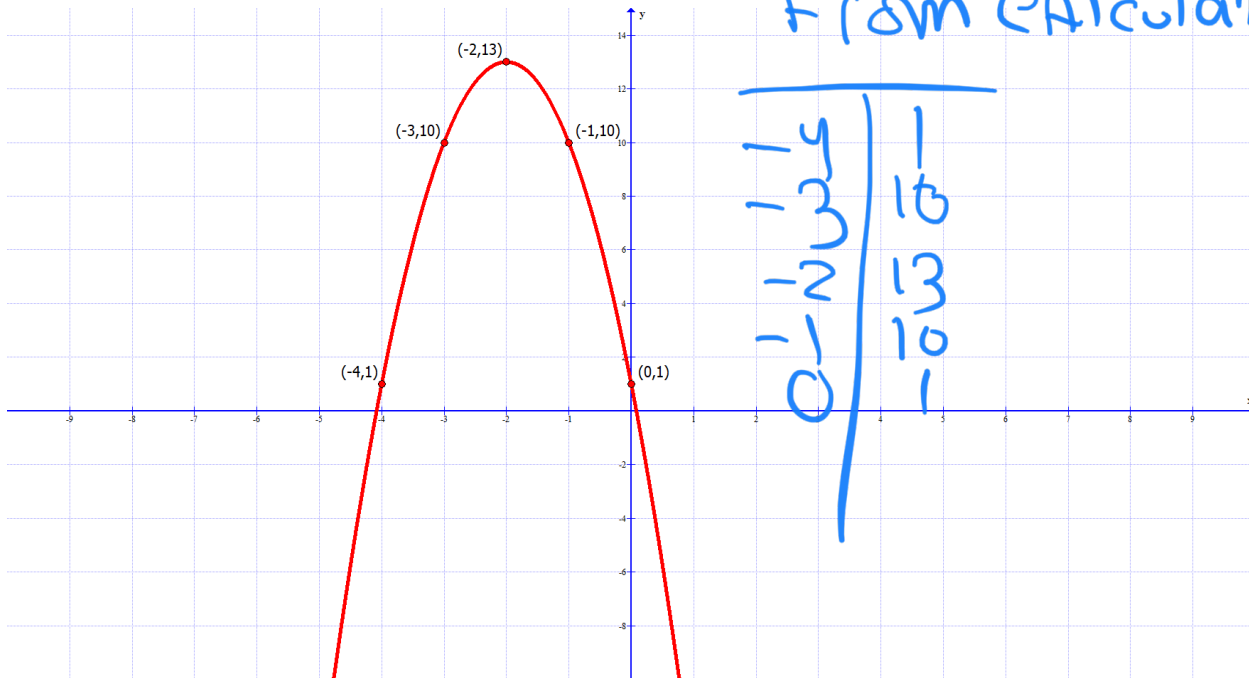
23) $f(x) = -3x^2 - 12x + 1$

a) Use completing the square to rewrite the problem in standard form

b) Describe the transformation as compared to the function $f(x) = x^2$

STretched, reflect x-AXIS, LEFT 2, UP 11

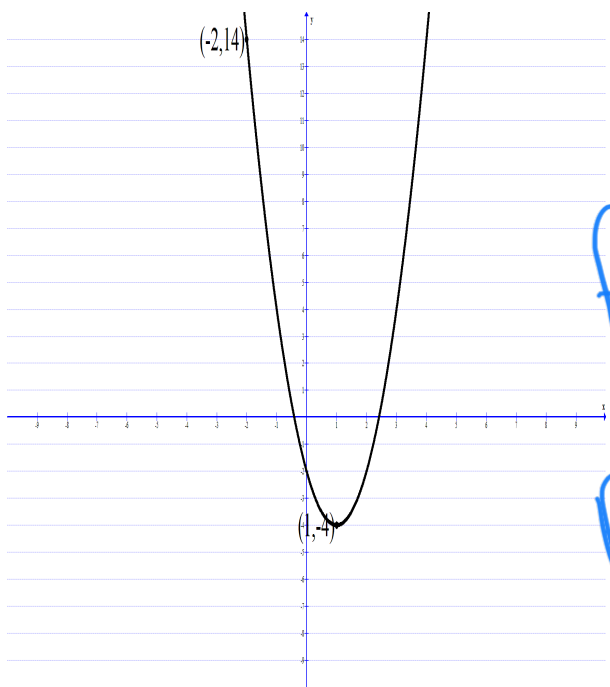
c) Sketch a graph, make sure to label the vertex. You may use your calculator, instead of making a table of values to create your graph



23a) $f(x) = -3(x^2 + 4x + c) + 1 + 3c$
 $c = (\frac{4}{2})^2 = (2)^2 = 4$
 $f(x) = -3(x^2 + 4x + 4) + 1 + 3(4)$
 $f(x) = -3(x+2)^2 + 1 + 12$
 $f(x) = -3(x+2)^2 + 13$

#25 – 32, determine the equation of the quadratic function

25)



$$f(x) = a(x-h)^2 + k$$

$$h = 1 \quad k = -4$$

$$f(x) = a(x-1)^2 + (-4)$$

$$f(x) = a(x-1)^2 - 4$$

Point $(-2, 14)$

x $f(x)$

$$14 = a(-2-1)^2 - 4$$

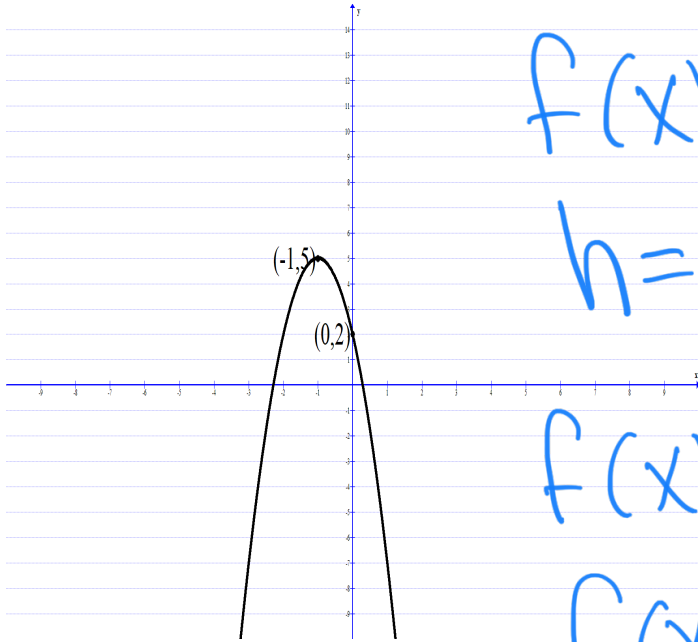
$$14 = a(-3)^2 - 4$$

$$14 = 9a - 4$$

$$18 = 9a$$
$$2 = a$$

$$f(x) = 2(x-1)^2 - 4$$

27)



$$f(x) = a(x-h)^2 + k$$

$$h = -1 \quad k = 5$$

$$f(x) = a(x - (-1))^2 + 5$$

$$f(x) = a(x+1)^2 + 5$$

$(0, 2)$

$$2 = a(0+1)^2 + 5$$

$$2 = a(1) + 5$$

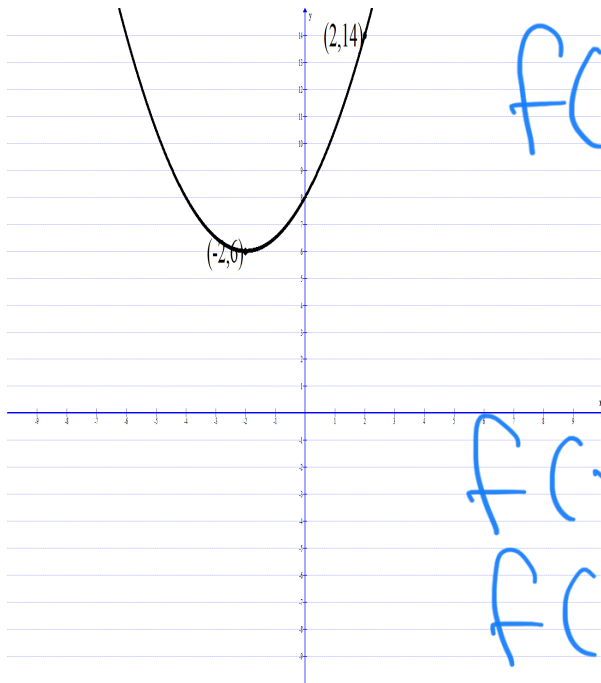
$$2 = a + 5$$

$$\begin{array}{r} -5 \quad -5 \\ \hline \end{array}$$

$$-3 = a$$

$$f(x) = -3(x+1)^2 + 5$$

29)



$$f(x) = a(x-h)^2 + k$$

$$h = -2 \quad k = 6$$

$$f(x) = a(x - (-2))^2 + 6$$

$$f(x) = a(x+2)^2 + 6$$

$$\begin{array}{l} (2, 14) \\ x \nearrow \quad \nearrow f(x) \end{array}$$

$$14 = a(2+2)^2 + 6$$

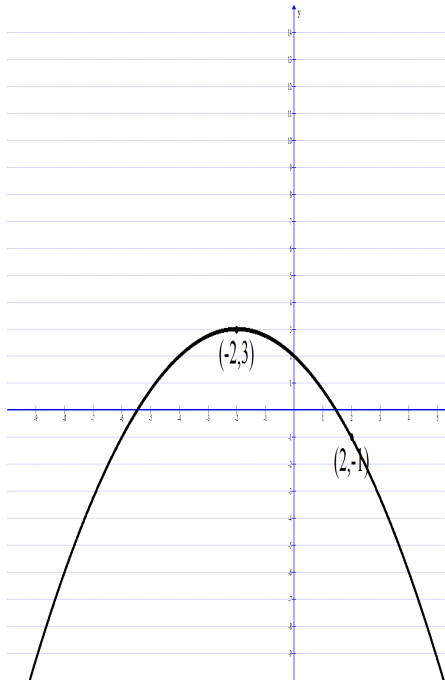
$$14 = 16a + 6$$

$$\begin{array}{r} -6 \qquad \qquad -6 \\ \hline \end{array}$$

$$\begin{array}{r} 8 = 16a \\ \frac{8}{16} = \frac{16a}{16} \\ \frac{1}{2} = a \end{array}$$

$$f(x) = \frac{1}{2}(x+2)^2 + 6$$

31)



$$f(x) = a(x-h)^2 + k$$

$$h = -2 \quad k = 3$$

$$f(x) = a(x - (-2))^2 + 3$$

$$f(x) = a(x + 2)^2 + 3$$

$$(2, -1)$$

$$\begin{array}{c} \uparrow \quad \uparrow \\ x \quad f(x) \end{array}$$

$$\begin{array}{r} -1 \\ -3 \end{array} = \begin{array}{r} a(2+2)^2 + 3 \\ -3 \end{array}$$

$$-4 = 16a$$

$$-\frac{4}{16} = a$$

$$-\frac{1}{4} = a$$

$$f(x) = -\frac{1}{4}(x+2)^2 + 3$$